FIELDS and WAVES in MODERN RADIO

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verified by differentiating:

$$\frac{\partial V}{\partial z} = -\frac{1}{v}F'\left(t - \frac{z}{v}\right) \qquad (4)$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{v^2}F''\left(t - \frac{z}{v}\right)$$

derivatives with respect to the entire
ing the two equations (4), (1) is

hat is meant by the statement that
This may be done by recognizing that
rence value of the function (i.e., keep
ument, $t - (z/v)$, a constant. This
e positive z direction with velocity v

$$-\frac{z}{v} = K$$

$$vt - Kv \qquad (5)$$

d degree differential equation has been
be written as any function of $t + (z/v)$
similar to those used for the first solu-
ve traveling in the negative z direction
lution to (1) is then

$$-\frac{z}{v}\Big) + F_2\left(t + \frac{z}{v}\right) \qquad (6)$$

$\cos \omega[t - (z/v)]$, $e^{j\omega[t-(z/v)]}$, and $[t + (z/v)]^3$ to

$[t - (z/v)]$ versus $\omega z/v$ for values of $\omega t = 0$,
nonstrates the interpretation as a propagating

GE AND CURRENT IN THE IDEAL LINE

developed above, Eq. 1·17(6), is sub-
e equation, 1·16(3),

$$'\left(t - \frac{z}{v}\right) + \frac{1}{v}F_2'\left(t + \frac{z}{v}\right) \qquad (1)$$

This expression may be integrated partially with respect to t:

$$I = \frac{1}{Lv}\left[F_1\left(t - \frac{z}{v}\right) - F_2\left(t + \frac{z}{v}\right)\right] + f(z) \qquad (2)$$

If this result were substituted in the other transmission line equation, 1·16(4), it would be found that the function of integration, $f(z)$, could only be a constant. But we are not interested in possible superposed d-c solutions in studying the wave solution, so this will be ignored. Equation (2) may then be written

$$I = \frac{1}{Z_0}\left[F_1\left(t - \frac{z}{v}\right) - F_2\left(t + \frac{z}{v}\right)\right] \qquad (3)$$

where

$$Z_0 = Lv = \sqrt{\frac{L}{C}} \qquad (4)$$

The constant Z_0 as defined by (4) is called the *characteristic impedance* of the line, and is seen from (3) to be the ratio of voltage to current for a single one of the traveling waves at any given point and given instant. The negative sign for the negatively traveling wave would of course be expected since the wave propagates to the left, and by our convention current is positive if flowing to the right.

1·19 REFLECTION AND TRANSMISSION AT A DISCONTINUITY

Most transmission line problems are concerned with junctions between a given uniform line and a line of different characteristic impedance, a load impedance or some other element that introduces a discontinuity. By Kirchhoff's laws, total voltage and current must be continuous across the discontinuity. The total voltage in the line may be regarded as the sum of voltage in a positively traveling wave, equal to V_1 at the point of discontinuity, and a voltage in a reflected or negatively traveling wave, equal to V_1' at the discontinuity. The sum of V_1 and V_1' must be V_L, the voltage appearing across the load impedance Z_L:

$$V_1 + V_1' = V_L \qquad (1)$$

Similarly, the sum of currents in the positively and negatively traveling waves of the line, at the point of discontinuity, must be equal to the current flowing into Z_L:

$$I_1 + I_1' = I_L \qquad (2)$$

By utilizing the relations between voltage and current for the two traveling waves as found in the preceding article, (2) becomes

$$\frac{V_1}{Z_0} - \frac{V_1'}{Z_0} = \frac{V_L}{Z_L} \tag{3}$$

By eliminating between (1) and (3), the ratio of voltage in the reflected wave to that in the incident wave (*reflection coefficient*) and the ratio of the voltage in the load to that in the incident wave (*transmission coefficient*) may be found:

$$\rho = \frac{V_1'}{V_1} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{4}$$

$$\tau = \frac{V_L}{V_1} = \frac{2Z_L}{Z_L + Z_0} \tag{5}$$

The most interesting, and perhaps the most obvious, conclusion from the above relations is this: there is no reflected wave if the terminating impedance is exactly equal to the characteristic impedance of the line. All energy of the incident wave is then transferred to the load impedance, which cannot be distinguished from a line of infinite length and characteristic impedance $Z_0 = Z_L$.

Note that for arbitrary time functions the "impedance" used above should be a pure resistance in order for the ratio of instantaneous load voltage to instantaneous load current to be given by Z_L, as used above. The impedance form has, however, been utilized so that the equation will apply immediately to steady-state sinusoidal waves expressed in the complex form for later use.

PROBLEMS

1·19*a* For arbitrary time functions, assuming Z_L a pure resistance, find the fraction of the incident power reflected, and the fraction of the incident power transmitted to Z_L.

1·19*b* Calculate the reflection coefficient, transmission coefficient, and the power quantities of Prob. *a* for $Z_L = 0$, $\frac{1}{2}Z_0$, Z_0, $2Z_0$, and ∞.

1·20 SOME SIMPLE PROBLEMS ON TRAVELING WAVES

A. D-C Voltage Applied to an Infinite Line. Consider the case of a d-c voltage V, suddenly applied to an ideal line of infinite length (Fig. 1·20*a*). The line starts to charge to voltage V, the wave front traveling with the velocity $v = 1/\sqrt{LC}$. Since there is never any discontinuity, there is never any reflected wave, and the only current is

en voltage and current for the two
›receding article, (2) becomes

$$\frac{V_1'}{Z_0} = \frac{V_L}{Z_L} \quad (3)$$

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e (*reflection coefficient*) and the ratio of
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$$\frac{'}{_1} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (4)$$

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ON TRAVELING WAVES

:n Infinite Line. Consider the case of a
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to charge to voltage V, the wave front
: $1/\sqrt{LC}$. Since there is never any dis-
reflected wave, and the only current is

that flowing in the positive wave, V/Z_0. This then is a d-c current flowing to the charges which appear on the line as voltage moves along. At any time t after the voltage is impressed, there is voltage V and current V/Z_0 in the line up to the point $z = vt$, and no voltage or current beyond.

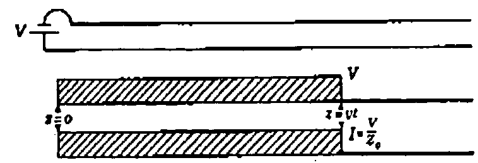

Fig. 1·20*a* Direct-current voltage suddenly applied to an infinite line.

B. D-C Voltage Applied to a Shorted Line. Suppose that the d-c voltage is applied to a line which is not infinite in length, but is shorted at some point, $z = l$ (Fig. 1·20*b*). We know that finally infinite current will flow if V is maintained. However, the mechanism of current build-up is interesting. After voltage is applied to the line, everything proceeds as in *A* until the time that the wave reaches the short

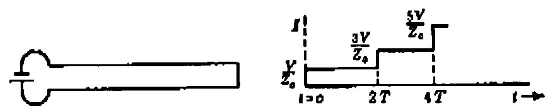

Fig. 1·20*b* Direct-current voltage suddenly applied to a shorted line.

circuit. At the time the incident wave with voltage V appears across the short circuit, which demands zero voltage, a reflected or negatively traveling wave of voltage $-V$ is sent back so that the sum of voltages in the two waves is indeed zero. Since current in the negative traveling wave is the negative of voltage divided by Z_0, this is $-(-V/Z_0)$ or $+V/Z_0$ and so adds directly to the current in the positive traveling wave. This reflected wave then moves to the left, leaving a wake of zero voltage and a current equal to $2V/Z_0$ behind it. As soon as the reflected wave has traveled back to the source, it brings the zero voltage condition back to this point so that the d-c voltage must send out a new wave of voltage V down the line, with associated current V/Z_0, making a total current in the line $3V/Z_0$ at this time. Current then builds up to infinity in the step manner indicated by Fig. 1·20*b*. T is the time l/v required for a wave to travel one way down the line.

C. Charged Line Connected to a Resistor. Consider an ideal line of length l initially charged to a d-c potential V, with a resistance R connected across the input at time $t = 0$.

The voltage across the resistance is the sum of the d-c voltage of the line and the voltage in the wave, V_1:

$$V_R = V + V_1 \tag{1}$$

The current flowing into the resistor is merely the negative of current for the positively traveling wave:

$$I_R = -I_1$$

or

$$\frac{V_R}{R} = -\frac{V_1}{Z_0} \tag{2}$$

By combining (1) and (2),

$$V_R = V\left(\frac{R}{R + Z_0}\right) = -V_1 \frac{R}{Z_0} \tag{3}$$

For example, if $R = Z_0$, the voltage appearing across the resistance at the first instant is half the d-c voltage of the line, as is the voltage appearing in the traveling wave. When this wave reaches the open

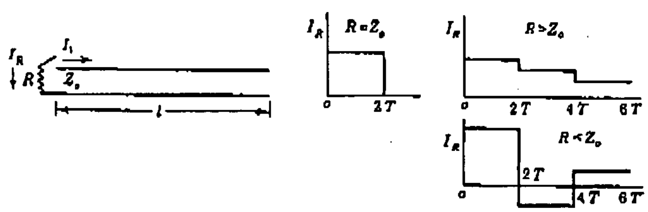

Fig. 1·20c Charged line of length l suddenly connected to a resistor.

end, there must then be started a reflected wave such that total current is zero, so current in the reflected wave must be $-I_1$ or $V/2R$. Because current in the reflected wave is the negative of voltage divided by Z_0, this will require a voltage $-V/2$ for the reflected wave. Thus in the case of $R = Z_0$, the original wave wipes out half the voltage, and the corresponding current, $-V/2Z_0$, is that which flows through R. The reflected wave wipes out the remaining half of the voltage and, of course, reduces current to zero. When this wave reaches $R = Z_0$ there is no further reflection, so all is still. Current wave shape is shown in Fig. 1·20c. Also shown are currents for $R > Z_0$ and $R < Z_0$.

PROBLEMS

1·20a An ideal line of length l is charged to d-c voltage V and shorted at its input at time $t = 0$. Sketch the current wave shape through the short as a function of time.

1·20b An ideal open-circui
voltage V suddenly applied a
through the input source as a

1·21 IDEAL LINE WITH APP

Much of the preceding
the type of variation with
mission lines. Most prac
least partially with sinuso
sinusoidal in time is appli
exponential (see Art. 1·11)

$V|$

Then the corresponding po

Similarly, a negatively tra

Or the total solution, ma
waves, is

$V = e^{j\omega t}[$

The corresponding current,

$I = \frac{e^{j\omega t}}{Z_0}[$

For problems in which we
soidal quantities, it is not n
each time, since it will alwa
plied by this factor; we rew

$V = V$

$I = \frac{1}{Z}$

where $\beta = \frac{\omega}{v}$

The quantity β is called the p
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if voltage and current are ob
exactly the same at points s